1. Geometric Construction

You can only used compass and straightedge which has no measure

Basic construction of lines.

Problem 1.1. You are given a line segment with end points A and B, draw a line bisecting AB and perpendicular to AB.

This line is called the *perpendicular bisector* of AB, and the point these two lines meet is called the *midpoint* of AB.

Problem 1.2. You are given an angle $\angle ABC$, find a point X in the interior of $\angle ABC$ such that $\angle ABX = \angle CBX$ and draw the line BX.

The line BX is called the *angle bisector* of $\angle ABC$.

The four special points of triangle. In this part, you are given a triangle $\triangle ABC$.

We will consider the intersection of the following two kind of lines, the medians and the altitudes.

It turns out that every lines of the same kinds will meet at a unique point.

Definition. Let M_{AB} be the midpoint of AB, draw the line passing through M_{AB} and C.

Simlarly, draw the others two lines.

These three lines are called the *median* of $\triangle ABC$.

Definition. Draw a line passing through A and perpendicular to BC, similar the other two lines.

These three lines are called the *altitude* of $\triangle ABC$.

Theorem. *The three medians of $\triangle ABC$ are concurrent.*

The three altitudes of $\triangle ABC$ are concurrent.

The three perpendicular bisectors of $\triangle ABC$ are concurrent.

The three angle bisectors of $\triangle ABC$ are concurrent.

Definition. The point that the three medians meet is called the *centroid* of $\triangle ABC$.

Definition. The point that the three altitudes meet is called the *orthocenter* of $\triangle ABC$.

Problem 1.3. Locate the center of the circle passing through the points A, B and C, and draw the circle.

The circle is called the *circumcircle* of $\triangle ABC$, and the center is called the *circumcenter*.

Problem 1.4. Locate the center of the circle which touches each side of the triangle, and draw the circle.

The circle is called the *incircle* of $\triangle ABC$, and the center is called the *incenter*.

Problem 1.5. Let G be the orthocenter of the triangle, $\angle BAC = x^\circ$, find $\angle BGC$ in terms of x.