

Chapter 2 Logics

In mathematics, all **statement** and **propositions** have its **truth value**, either **true** or **false**. To prove a formula is valid, we have to logically formulate many and many statements to show the formula is **true**.

Those **truth value** are also called **Boolean value**.

Examples:

- (1) It is raining now. (It can either be **true** or **false**.)
- (2) $1+1=3$. (**false**)
- (3) Any number whose digit sum is divisible by 9 must be divisible by 9. (**true**)
- (4) There are infinitely many prime numbers. (**true**)
- (5) $1 > 3$ and $2 > 1$. (**false**)

Proof of (4):

Suppose there are only finitely many prime numbers, say n .

Let the n primes be p_1, p_2, \dots, p_n .

Let $x = p_1 p_2 \dots p_n + 1$, which is an integer.

Then for any p_i , x is not divisible by p_i , since they leave a remainder 1 when divided by p_i .

Then x is a new prime number, contradiction.

Logical operators:

As for numbers there are additions, subtractions etc...

Also, there are operations for **Boolean value**.

not: (precedence 1)

A	not A
True	False
False	True

and: (precedence 2)

A	B	A and B
True	True	True
True	False	False
False	True	False
False	False	False

or: (precedence 3)

A	B	A or B
True	True	True
True	False	True
False	True	True
False	False	False

Conditional statements:

A **conditional statement** consists of two parts, the **if-part** and the **then-part**.
A **conditional statement** has a truth value **false** only when the **if-part** is **true** but the **then-part** is **false**, otherwise it is considered to be a **true** statement.

*Note: You can just treat the **truth value** as indicating whether it is telling lies.
Therefore, when the **if-part** is **false**, we have no way to test it, so we must trust it.
We also say “A implies B” for the statement “if A then B”.*

Examples:

- (1) If this year is a leap year, then there are 366 days. (true)
- (2) If $x = 2$, then $x^2 = 4$. (true)
- (3) If $x \neq 2$, then $x^2 \neq 4$. (false)
- (4) If $x^2 \neq 4$, then $x \neq 2$. (false)
- (5) If $x^2 = 4$, then $x = 2$. (false) (Consider the case $x = -2$)
- (6) If $1 > 2$, then the Earth is a triangle. (true) (Why is it true???)
- (7) If there are 366 days, then this year is a leap year. (true)
- (8) If $x - 3 = 4$, then $x = 7$. (true)

Facts:

$A \text{ or not } A \equiv \text{true}$	$A \text{ and not } A \equiv \text{false}$
$A \text{ and } A \equiv A \text{ or } A \equiv A$	
$\text{If } A \text{ then } B \equiv A \text{ and not } B \equiv \text{not } A \text{ or } B \equiv \text{If not } B \text{ then not } A.$	

If and only if:

From (2) and (4), we can see that the order of a **conditional statement** is important.
Also, from (2) and (3), we also know that we cannot directly apply the **not** operator.
So, you can see that an **if-then** statement is single direction only.

But, for statements like (8) or the pair (1) and (7), you know it is in both directions.
We will use the words “**if and only if**” (*iff*) to represent such situation.

Examples:

- (1) A number is rational **if and only if** it can be represented as a fraction of two integers.
- (2) An integer is divisible by 3 **if and only if** its digit sum is divisible by 3.

End of Chapter