

## Chapter 0                      Functions

### **Intuitive meaning of function:**

In mathematics, a **function** is something that returns a corresponding **output** when you give it an **input**.

We usually denote a **function** by small letter  $y, f, g, h, \dots$

Mathematically, we use small letter  $x, y, z$  to represent the **input**, for single input the corresponding output is denoted by  $y(x), f(x), g(x), h(x), \dots$ .

In case of double inputs, we denote the output by  $f(x, y)$ .

Similarly for triple ones, we denote by  $f(x, y, z)$ .

For  $n$  inputs, we denote by  $f(x_1, x_2, x_3, \dots, x_n)$ .

### Examples:

1) If we define a function  $f$  to be  $f(x) = x^2 + x + 1$ , then we have  $f(1) = 1^2 + 1 + 1 = 3$ .

This time, the **input** is 1 and the **output**  $f(1)$  is 3.

2)  $f(x) = x + 10^x$ , then  $f(2) = 2 + 10^2 = 102$ . What is  $f(3)$  and  $f(2+3)$ ?

*CAUTION: in the above examples  $f(x+y) \neq f(x) + f(y)$  and  $f(xy) \neq f(x)f(y)$ .*

3)  $f(x, y) = 10^{x+2y}$ , then  $f(1, 2) = 10^{1+2(2)} = 100000$ .

In this case, the **input** is a pair of number (1,2)

By observation, we know that the **output** depends on the **input**, so the **input**  $x$  is also called **variable**, just like the *unknown variable*  $x$  when solving equation.

### Special functions learnt in lower form:

Those  $\sin, \cos, \tan, \sin^{-1}, \cos^{-1}, \tan^{-1}$  are called **trigonometric function**.

\*WE DON'T HAVE  $\sin 90^\circ = \sin 45^\circ + \sin 45^\circ$  etc.\*

*In fact  $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ , you can use your calculator to try this.*

*If time permitted, I will explain why the formula is true later on.*

Another kind of **function**  $a^x$  ( $a$  to the  $x^{\text{th}}$  power) is called **exponential function**.

This kind of functions satisfies the rule  $f(x+y) = f(x)f(y)$ .

### Question:

1) Is there any function satisfying the rule  $f(xy) = f(x) + f(y)$ ?

2) What is the meaning of  $2^{0.5}, 2^{\sqrt{2}}, 2^\pi, \pi^\pi$ ? How to calculate them?

\*The answer to the two questions is related to **logarithmic function**.\*

**Chapter 0 Exercises**

(You are not allowed to use any calculator)

**Find the followings:**

1.  $f(x) = x^2 - 2x + 1$

- |                |       |                  |       |
|----------------|-------|------------------|-------|
| (a) $f(-1)$    | _____ | (b) $f(0)$       | _____ |
| (c) $f(f(-1))$ | _____ | (d) $f(f(0))$    | _____ |
| (e) $f(x+1)$   | _____ | (f) $f(f(x))$    | _____ |
| (g) $f(x^2)$   | _____ | (h) $f(f(f(x)))$ | _____ |

$f(f(x))$  is called **composition** of **function**  $f$ .

2. It is also known that  $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ .

Show that  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan(x)\tan(y)}$ .

3.  $g(x) = \frac{1}{x-4}$

- |                 |       |                                 |       |
|-----------------|-------|---------------------------------|-------|
| (a) $g(-1)$     | _____ | (b) $g(0)$                      | _____ |
| (c) $g(5)$      | _____ | (d) $g\left(\frac{2}{9}\right)$ | _____ |
| (e) $g(x+4)$    | _____ | (f) $g\left(\frac{1}{x}\right)$ | _____ |
| (g) $g(g(x+4))$ | _____ | (h) $g(g(x))$                   | _____ |

**End of Chapter**